

# Infrared Behaviour of Propagators and Vertices

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## Abstract

We elucidate constraints imposed by confinement and dynamical chiral symmetry breaking on the infrared behaviour of the dressed-quark and -gluon propagators, and dressed-quark-gluon vertex. In covariant gauges the dressing of the gluon propagator is completely specified by  $\mathcal{P}(k^2) := 1/[1 + \Pi(k^2)]$ , where  $\Pi(k^2)$  is the vacuum polarisation. In the absence of particle-like singularities in the dressed-quark-gluon vertex, extant proposals for the dressed-gluon propagator that manifest  $\mathcal{P}(k^2 = 0) = 0$  and  $\max(\mathcal{P}(k^2)) \sim 10$  neither confine quarks nor break chiral symmetry dynamically. This class includes all existing estimates of  $\mathcal{P}(k^2)$  via numerical simulations.

*Key words:* Gluon and quark Schwinger functions; Dynamical Chiral Symmetry Breaking; Confinement; Dyson-Schwinger equations; Lattice-QCD

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Strong interaction phenomena are characterised by dynamical chiral symmetry breaking (DCSB) and colour confinement. At low energy, DCSB is the more important; for example, in its absence the  $\pi$ - and  $\rho$ -mesons would be nearly degenerate and at the simplest observational level that would lead to a markedly different line of nuclear stability. These phenomena can be related to the infrared behaviour of elementary Schwinger functions in QCD and herein we elucidate some constraints they place on this behaviour.

As described pedagogically in Ref. [1], DCSB can be studied using the QCD “gap equation”; i.e., the Dyson-Schwinger equation (DSE) for the renormalised dressed-quark propagator (connected, 2-point, dressed-quark Schwinger function),  $S(p)$ :

$$S(p)^{-1} = i\gamma \cdot p A(p^2) + B(p^2) \equiv \frac{1}{Z(p^2)} [i\gamma \cdot p + M(p^2)] \quad (1)$$

$$= Z_2(i\gamma \cdot p + m_{\text{bm}}) + Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \gamma_\mu S(q) \Gamma_\nu(q, p), \quad (2)$$

where  $D_{\mu\nu}(k)$  is the renormalised dressed-gluon propagator,  $\Gamma_\mu(q; p)$  is the renormalised dressed-quark-gluon vertex, and  $\int_q^\Lambda := \int^\Lambda d^4q/(2\pi)^4$  represents mnemonically a translationally-invariant regularisation of the integral, with  $\Lambda$  the regularisation mass-scale. The final stage in any calculation is to take the limit  $\Lambda \rightarrow \infty$ . In (1),  $Z_1$  and  $Z_2$  are the renormalisation constants for the quark-gluon vertex and quark wave function, and  $m_{\text{bm}}$  is the current-quark bare mass: the chiral limit is obtained with  $m_{\text{bm}} = 0$  [2,3].

This equation is relevant because an order parameter for DCSB is the chiral-limit, vacuum quark condensate [2]:

$$\langle \bar{q}q \rangle_\mu^0 := Z_4(\mu^2, \Lambda^2) N_c \int_q^\Lambda \text{tr}_D [S_{\hat{m}\alpha m_{\text{bm}}=0}(q)], \quad (3)$$

where  $Z_4(\mu^2, \Lambda^2) = [\alpha(\Lambda^2)/\alpha(\mu^2)]^{\gamma_m(1+\xi/3)}$  at one-loop order, with  $\mu^2$  the renormalisation point,  $\xi$  the covariant-gauge fixing parameter [ $\xi = 0$  specifies Landau gauge] and  $\gamma_m = 12/(33 - 2N_f)$  the gauge-independent mass anomalous dimension. The  $\xi$ -dependence of  $Z_4(\mu^2, M^2)$  is just that required to ensure that  $\langle \bar{q}q \rangle_\mu^0$  is gauge independent. It follows from (1) that an equivalent order parameter is  $\mathcal{X} := B(p^2 = 0)$ . Chiral symmetry is dynamically broken when, with  $m_{\text{bm}} = 0$ ,  $\mathcal{X} \neq 0$ .

Equation (2) is a nonlinear integral equation and the properties of its solution depend on the kernel, which is constructed from  $D_{\mu\nu}(k)$  and  $\Gamma_\mu(q, p)$ . As summarised in Ref. [4], the connected, dressed-gluon 2-point function,  $D_{\mu\nu}(k)$ , satisfies an oft analysed DSE. The qualitative conclusion of these DSE studies is that if the ghost-loop in the gluon DSE is unimportant, which is tautological in the ghostless axial gauges, then relative to the free gauge boson propagator the dressed-gluon propagator is significantly enhanced in the vicinity of  $k^2 = 0$ , where it is a regularisation of  $1/k^4$  as a distribution [5]. That enhancement persists to  $k^2 \sim 1\text{-}2 \text{ GeV}^2$ , where a perturbative analysis becomes quantitatively reliable.

The other term in the kernel of (1) is  $\Gamma_\mu(q, p)$ , the connected, irreducible, renormalised dressed-quark-gluon vertex:  $p, q$  are the momentum labels of the quark and antiquark lines, and the total momentum  $P := p - q$ . The analogue of this vertex in QED has been much studied and it is argued [6] that it should

not exhibit particle-like singularities at  $P^2 = 0$ .<sup>1</sup> The reasoning is simple: such singularities do not arise at low order in perturbation theory and hence such a vertex contradicts perturbation theory in any domain on which a weak coupling expansion is valid.

Another arguably stronger reason is that a singularity of this type signals the existence of a massless bound state that mixes with the gauge boson, and such states have not been observed. This feature can be elucidated by considering the colour-singlet, axial-vector vertex in QCD, which is the solution of

$$\left[\Gamma_{5\mu}^H(k; P)\right]_{tu} = Z_2 \left[\gamma_5 \gamma_\mu \frac{T^H}{2}\right]_{tu} + \int_q^\Lambda [\chi_{5\mu}^H(q; P)]_{sr} K_{tu}^{rs}(q, k; P), \quad (4)$$

where the matrix  $T^H$  specifies the flavour structure of the vertex,  $\chi_{5\mu}^H(q; P) := \mathcal{S}(q_+) \Gamma_{5\mu}^H(q; P) \mathcal{S}(q_-)$  with  $q_+ := q + \eta_P P$ ,  $q_- := q - (1 - \eta_P)P$  and  $P$  the total momentum, and  $\mathcal{S}(q) := \text{diag}[S_u(q), S_d(q), S_s(q), \dots]$ . In (4),  $K$  is the fully-amputated, quark-antiquark scattering kernel: by definition it does not contain quark-antiquark to single gauge-boson annihilation diagrams, such as would describe the leptonic decay of the pion, nor diagrams that become disconnected by cutting one quark and one antiquark line.

In the chiral limit the solution of this equation is [3]

$$\begin{aligned} \Gamma_{5\mu}^H(k; P) = & \frac{T^H}{2} \gamma_5 \left[ \gamma_\mu F_R(k; P) + \gamma \cdot k k_\mu G_R(k; P) - \sigma_{\mu\nu} k_\nu H_R(k; P) \right] \\ & + \tilde{\Gamma}_{5\mu}^H(k; P) + f_H \frac{P_\mu}{P^2} \Gamma_H^5(k; P), \end{aligned} \quad (5)$$

where:  $F_R$ ,  $G_R$ ,  $H_R$  and  $\tilde{\Gamma}_{5\mu}^H$  are regular as  $P^2 \rightarrow 0$ ;  $P_\mu \tilde{\Gamma}_{5\mu}^H(k; P) \sim \mathcal{O}(P^2)$ ; and  $\Gamma_H^5(k; P)$  is the Bethe-Salpeter amplitude for a massless pseudoscalar bound state; i.e.,  $\Gamma_H^5(k; P)$  satisfies the associated homogeneous Bethe-Salpeter equation. The vertex is gauge covariant: the pole-position and  $f_H$ , which is the leptonic decay constant, are gauge *invariant* [2] and the bound state amplitude responds in a well-defined manner to a gauge transformation. In 3-flavour, massless QCD the poles in the axial-vector vertices correspond to the octet of Goldstone bosons. There should be no such singularities in the colour-singlet vector vertex, and this is verified in model studies [7].

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<sup>1</sup> A particle-like singularity is one of the form  $(P^2)^{-\alpha}$ ,  $\alpha \in (0, 1]$ . In this case one can write a spectral decomposition for the vertex in which the spectral densities are non-negative. This is impossible if  $\alpha > 1$ .  $\alpha = 1$  is the ideal case of an isolated,  $\delta$ -function singularity in the spectral densities and hence an isolated, free-particle pole.  $\alpha \in (0, 1)$  corresponds to an accumulation, at the particle pole, of branch points associated with multiparticle production.

Similar observations apply to the fully-amputated, dressed-quark-gluon vertex,  $\Gamma_\mu(q, p)$ . It satisfies an integral equation like (4) with the complication that, in addition to the term involving  $K$ , there are 3 other terms involving the scattering kernels for:  $q\bar{q}$  to 2-gluon,  $K^{2g}$ ;  $q\bar{q}$  to ghost-antighost,  $K^{ghgh}$ ; and  $q\bar{q}$  to 3-gluon,  $K^{3g}$ . Recall that, by definition, none of these kernels contain single-gluon intermediate states. Hence a massless, particle-like singularity in this vertex signals the presence of a colour-octet bound state in one of the scattering matrices:  $M := K/[1 - (SS)K]$ ;  $M^{2g} := K^{2g}/[1 - (DD)K^{2g}]$ ; etc. As no such coloured bound states have been observed, one must reject calculations or Ansätze for any of the Schwinger functions that entail a particle-like singularity in this vertex. The same objection applies to particle-like singularities in the fully-amputated, dressed-3-gluon vertex, and all like  $n$ -point functions. This anticipates the result of an estimate [8] of the 3-gluon vertex via a numerical simulation of lattice-QCD, which shows no evidence for a singularity of any kind.

From a phenomenological perspective, a combination of  $D_{\mu\nu}(k)$  enhanced as described and  $\Gamma_\mu(q, p)$  without particle-like singularities is an excellent result, since it is *sufficient* to yield DCSB and confinement<sup>2</sup> *without* fine-tuning [1]. It can also provide for a quantitatively accurate description of a wide range of hadronic observables [1,3,9], although this depends more on the detailed form of  $D_{\mu\nu}(k)$  and  $\Gamma_\mu(q, p)$ .

Does the phenomenology of the strong interaction *require* that the gluon propagator be strongly enhanced relative to the free gauge-boson propagator? In Landau gauge

$$D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Delta(k^2), \quad \Delta(k^2) := \frac{1}{k^2} \mathcal{P}(k^2), \quad (6)$$

and the question can be posed as: Do the observable phenomena *necessarily* require ( $\Lambda_{\text{QCD}}^{N_f=4} \sim 220 \text{ MeV}$ )

$$\mathcal{P}(k^2) \gg 1 \text{ for } 0 < k^2 \lesssim 10 \Lambda_{\text{QCD}}^2? \quad (7)$$

We do not have an answer but we can explore alternatives. The antithesis of

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<sup>2</sup> Herein confinement means that the dressed-quark and -gluon 2-point functions do not have a Lehmann representation; i.e., do not have a spectral representation with a non-negative spectral density. This is a sufficient but not necessary condition.[10]

(7) is the extreme<sup>3</sup> possibility that

$$\mathcal{P}(k^2 = 0) = 0, \quad \mathcal{P}(k^2) \leq 1 \quad \forall k^2, \quad (8)$$

which was canvassed in Ref. [11]. Therein the ghost-loop contribution to the gluon DSE is retained and the Ansätze for the 3-gluon and quark-gluon vertices exhibit *ideal* particle-like poles [ $\alpha = 1$ ]. Since these poles are an essential element of the solution procedure then, in the absence of a physically sensible interpretation or explanation of them, one could simply reject this result.

Alternatively, one can suppose that (8) is more robust than the procedure employed to motivate it and explore the phenomenological consequences of the conjecture [11]:  $\mathcal{P}_S(k^2) := k^4/(k^4 + b^4)$ , where  $b$  is a dynamically generated mass scale. Following this approach it was found that if there are no particle-like singularities in the quark-gluon vertex,  $\Gamma_\mu(q, p)$ , then  $\mathcal{P}_S(k^2)$  is unable to confine quarks [12,13] and  $b$  must be fine-tuned to very small values [ $b < b_c \simeq \Lambda_{\text{QCD}}$ ] if DCSB is to occur [12–14]. It is therefore apparent that (8) is phenomenologically difficult to maintain.<sup>4</sup>

Nevertheless, the hypothesis has been explored in studies [15] of the dressed-gluon 2-point function using numerical simulations of lattice-QCD.  $\mathcal{P}(k^2 = 0)$  is necessarily finite in simulations on a finite lattice because of the inherent infrared cutoff. Thus one can only truly determine  $\mathcal{P}(k^2 \sim 0)$  by considering the behaviour of the numerical result in both the countable limit of infinitely many lattice sites and the continuum limit. The form  $\mathcal{P}_S(k^2)$  does not provide as good a fit to the lattice data as an alternative form, which in the countable limit is

$$\mathcal{P}_L(k^2) := \frac{k^2}{M^2 + Z k^2 (k^2 a^2)^\eta}, \quad 0 < k^2 < 0.6/a^2 \sim 50 \Lambda_{\text{QCD}}^2, \quad (9)$$

where  $1/a \approx 2.0 \text{ GeV}$  is the inverse lattice spacing,  $Z \approx 0.1$ ,  $\eta \approx 0.53$ , and  $M \approx 0.16 \text{ GeV}$ . This takes the maximum value  $\mathcal{P}_L(k^2 = 21\Lambda_{\text{QCD}}^2) = 13.6$  and corresponds to a less extreme alternative to (7), which we shall characterise

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<sup>3</sup> “Extreme” because it corresponds to a screening of the fermion-fermion interaction, as familiar in an electrodynamical plasma, rather than the antiscreening often discussed in zero-temperature chromodynamics.

<sup>4</sup> DCSB requiring  $b \sim 0$  is indicative of the *dynamical evasion* of (8) since  $\mathcal{P}_S(k^2) \rightarrow 1$  rapidly for small values of  $b$ . We do not consider the possibility that an irreducible vertex has a non-particle-like singularity; i.e., a singularity of the form  $(k^2)^{-\alpha}$ ,  $\alpha > 1$ , as there is no indication of such behaviour in any study to date.

as<sup>5</sup>

$$\mathcal{P}(k^2 = 0) = 0, \max(\mathcal{P}(k^2)) \lesssim \mathcal{O}(10). \quad (10)$$

The feature  $\mathcal{P}(k^2 = 0) = 0$  is critically dependent on whether  $M$  is nonzero, or not. It appears to be nonzero in the countable limit but, as emphasised in Ref. [15], the behaviour of  $M$  (and  $\eta$ ) in the continuum limit is unknown.

The phenomenological implications of (9) can be explored using the methods of Ref. [12]. A preliminary estimate follows by observing that  $\mathcal{P}_L(k^2)$  is approximately equivalent to  $\mathcal{P}_S(k^2)$  if one identifies  $b_L \sim \sqrt{M/a} = 0.57 \text{ GeV}$ . Hence one expects that (9) does not generate DCSB nor confine quarks.<sup>6</sup> In order to quantitatively verify this conclusion we note that: it is the combination  $g^2 \mathcal{P}(k^2)/k^2$  that appears in (1) and  $g^2$  is not determined in Ref. [15]; and we must extrapolate  $\mathcal{P}_L(k^2)$  outside the fitted domain. Both of these requirements are fulfilled if we assume that a one-loop perturbative analysis is reliable for  $k^2 \gtrsim 25 \Lambda_{\text{QCD}}^2$ , and employ<sup>7</sup>

$$g^2 \mathcal{P}_l(k^2) := \begin{cases} g_m^2 \mathcal{P}_L(k^2), & k^2 \leq k_m^2 \\ g^2(k^2), & k^2 > k_m^2, \end{cases} \quad (11)$$

requiring that  $\Delta_l(k^2) := \mathcal{P}_l(k^2)/k^2$  and its first derivative be continuous at  $k_m^2$ . This procedure yields  $\Delta_l(k^2)$  in Fig. 1 with

$$g_m = 0.65, \quad k_m^2 = 30 \Lambda_{\text{QCD}}^2. \quad (12)$$

It is now straightforward to solve (1) with a variety of Ansätze for the quark-gluon vertex that do not exhibit particle-like singularities.<sup>8</sup> We employed the bare vertex  $\Gamma_\mu(p, q) := \gamma_\mu$ ; the Ansatz [6]:

<sup>5</sup> A dressed-gluon propagator satisfying (8) automatically satisfies (10). The model of Ref. [16] is in the class specified by (10), as are the fitted forms obtained in all existing lattice-QCD simulations.

<sup>6</sup> A value of  $b \approx 0.4 \text{ GeV} > b_c$  in  $\mathcal{P}_S(k^2)$  provides the best fit to the lattice data and this supports the same conclusion.

<sup>7</sup> In representing  $\mathcal{P}(k^2)$  in the ultraviolet via the running coupling,  $g^2(k^2)$ , we are effectively enforcing the identity:  $\tilde{Z}_1 = \tilde{Z}_3$  between the ghost-gluon-vertex and ghost wave function renormalisation constants. This “Abelian Approximation” is a phenomenologically well justified Ansatz that ensures [3] the correct one-loop anomalous dimension for both: the dressed-quark propagator obtained as a solution of (1); and the pseudoscalar meson bound state amplitudes. At most it introduces an error at large- $k^2$  in the power of the  $\ln$ -dependence of  $\mathcal{P}(k^2)$ .

<sup>8</sup> Equation (11) defines a renormalisable model quark DSE, which we solved in the manner described in Ref. [3]. For simplicity, we renormalised at the momentum

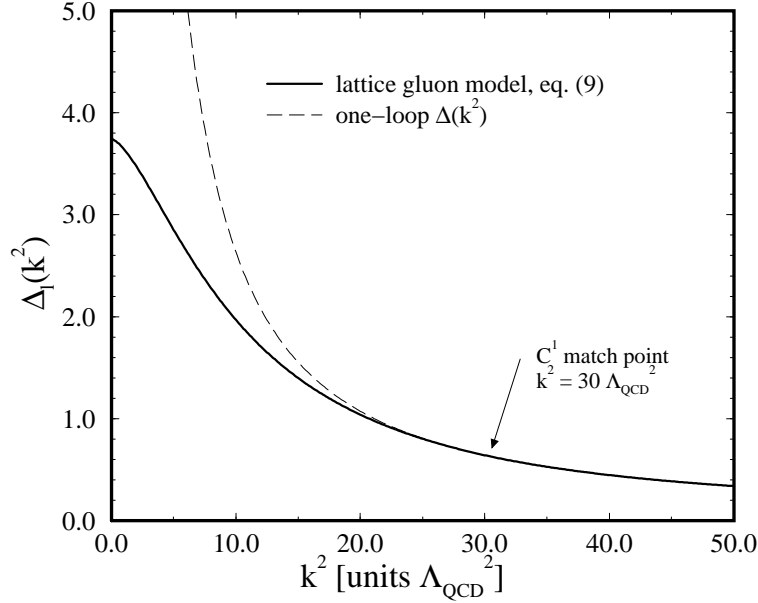


Fig. 1.  $\Delta_l(k^2) := \mathcal{P}_l(k^2)/k^2$  from (11).  $\mathcal{P}_l(k^2)$  is (9) in the infrared and extrapolates this lattice model outside the domain accessible in the simulation [15].

$$i\Gamma_\mu^{\text{BC}}(p, q) := i\Sigma_A(p^2, q^2) \gamma_\mu + (p + q)_\mu \left[ \frac{1}{2} i\gamma \cdot (p + q) \Delta_A(p^2, q^2) + \Delta_B(p^2, q^2) \right], \quad (13)$$

where  $\Sigma_A(p^2, q^2) := [A(p^2) + A(q^2)]/2$ ,  $\Delta_A(p^2, q^2) := [A(p^2) - A(q^2)]/[p^2 - q^2]$  and  $\Delta_B(p^2, q^2) := [B(p^2) - B(q^2)]/[p^2 - q^2]$ ; and an augmented form [17]

$$\Gamma_\mu^{\text{CP}}(p; q) := \Gamma_\mu^{\text{BC}}(p, q) + \Gamma_\mu^6(p, q), \quad (14)$$

$$\Gamma_\mu^6(p, q) := \frac{\gamma_\mu(p^2 - q^2) - (p + q)_\mu(\gamma \cdot p - \gamma \cdot q)}{2d(p, q)} [A(p^2) - A(q^2)], \quad (15)$$

with  $d(p, q) := ([p^2 - q^2]^2 + [M(p^2)^2 + M(q^2)^2]^2)/(p^2 + q^2)$ , each of which allows the quark DSE to be solved in isolation. In all cases we found  $\mathcal{X} = 0$  in the chiral limit; i.e., no DCSB with  $B(p^2) \equiv 0$ .

The absence of DCSB means it is straightforward to decide whether (11) generates confinement. In this case quark confinement is manifest if  $Z(p^2)$  is smooth and vanishes at  $p^2 = 0$ ; while the existence of a Lehmann representation and the concomitant lack of confinement is clear if  $Z(p^2)$  does not vanish at  $p^2 = 0$ . In Fig. 2 we plot the solution  $Z(p^2)$  obtained from (1) with the vertex Ansätze introduced above. The behaviour of the solution is qualitatively equivalent in

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cutoff,  $\Lambda_{\text{UV}} \sim 10^4 \Lambda_{\text{QCD}}$ , since the  $p^2$ -evolution of  $A(p^2)$  and  $B(p^2)$  beyond that point is completely determined by  $g^2(k^2)$ .

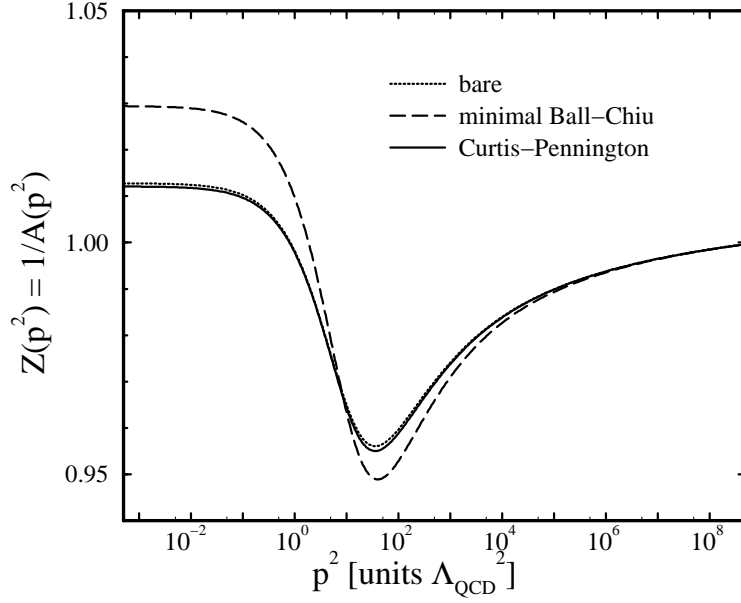


Fig. 2.  $Z(p^2)$  obtained as the solution of (1) using (11) with: (13) - solid line; (14) - dashed line; and  $\Gamma_\mu(p; q) = \gamma_\mu$  - dotted line. That (9) does not confine quarks is manifest in the result:  $Z(p^2 = 0) \neq 0$ , which is independent of the vertex Ansatz.

each case. This demonstrates explicitly that (9) neither generates DCSB nor confinement,<sup>9</sup> thereby confirming our preliminary hypothesis based on the correspondence with  $\mathcal{P}_S$  via an effective value of  $b$ .

We have also solved the equation for  $Z(p^2)$  using

$$\tilde{\mathcal{P}}_l(k^2) := \left(1 + \varsigma e^{-k^2/\Lambda_{\text{QCD}}^2}\right) \mathcal{P}_l(k^2) \quad (16)$$

where  $\varsigma$  is a variable “strength” parameter. Increasing  $\varsigma$  moves the peak in  $\tilde{\mathcal{P}}_l(k^2)$  toward  $k^2 = 0$  and increases its height, thereby making it increasingly like the model of Ref. [3]. The form of  $Z(p^2)$  is qualitatively unchanged and hence there is no signal for the onset of confinement until  $\varsigma \gtrsim 300$ . At  $\varsigma = 300$  the maximum value is

$$\tilde{\mathcal{P}}_l(k^2 = 0.98 \Lambda_{\text{QCD}}^2) = 210 \quad (17)$$

<sup>9</sup> The result of a recent numerical simulation [18] is pointwise smaller in magnitude ( $\lesssim 1/3$ ) than (9) on the entire fitted domain and hence the same conclusions apply in that case. The discrepancy is neither identified nor explained in Ref. [18]. Both simulations use  $\beta = 6.0$ . Ref. [15] has 500 configurations on a  $24^3 \times 48$  lattice with  $\langle \partial_\mu A_\mu(x) \rangle_{\text{Lat.}} < 10^{-6}$ , while Ref. [18] has 75 configurations on a  $32^3 \times 64$  lattice with  $\langle \partial_\mu A_\mu(x) \rangle_{\text{Lat.}} < 10^{-12}$ .



and  $\tilde{\mathcal{P}}_l(0.98 \Lambda_{\text{QCD}}^2)/\tilde{\mathcal{P}}_l(30 \Lambda_{\text{QCD}}^2) = 16$ , cf.  $\mathcal{P}_l(21 \Lambda_{\text{QCD}}^2)/\mathcal{P}_l(30 \Lambda_{\text{QCD}}^2) = 1.0$ . In the model of Ref. [3] the peak is at  $k^2 = 3.7 \Lambda_{\text{QCD}}^2$  and the value of this ratio is 44, neglecting only for the purpose of this comparison the purely long-range,  $\delta^4(k)$ -part of that interaction. A comparison of  $g_m^2 \tilde{\mathcal{P}}_l(\Lambda_{\text{QCD}}^2) \approx 89$  with the critical coupling of  $g_c^2 \approx 11$  in Refs. [19] shows that such large values of  $\varsigma$  ensure DCSB.

The hypothesis (8) has also re-emerged recently in a DSE study [20] that is qualitatively akin to Ref. [11]: in its result that  $\mathcal{P}(k^2) = A k^4$  for  $k^2 \simeq 0$ ; in postulating a significant role for the ghost-loop in the gluon propagator; and in employing a ghost-gluon vertex that is free of particle-like singularities.<sup>10</sup> Following the above analysis, the results of Refs. [12,13] can be applied directly in this case. The gluon propagator is smooth and can be characterised by a value of  $b^4 \approx \lambda^4/A$ , where  $A \sim 1$  and  $\lambda$  is a mass-scale that is left undetermined in Ref. [20]. Choosing any reasonable value of  $\lambda$ ; e.g.,  $\lambda \gtrsim \Lambda_{\text{QCD}}$ , this gluon propagator, with a quark-gluon vertex that is free of particle-like singularities, neither yields DCSB nor confines quarks.

It was recognised in Ref. [15] that the hypotheses (8) and (10) are problematic, and this is emphasised by our results: in the absence of particle-like singularities in the dressed-quark-gluon vertex, the possibility of a dressed-gluon propagator satisfying (10) is excluded by the existence of DCSB. In the lattice simulations the infrared scale,  $M$ , that entails (10) may vanish in the continuum limit: DCSB and confinement appear to require that. However, this limit is presently unexplored. In the DSE study the treatment of the ghost-gluon vertex and ghost-gluon scattering kernel is rudimentary and, with the information currently available to us, it is difficult to estimate the sensitivity of the results to the truncations. The incompatibility we identify between (8) and the phenomena of the strong interaction suggest that an examination of the effects of these truncations is necessary.

Our primary predicate is that the dressed-quark-gluon vertex should not exhibit a particle-like singularity. If this is false and the vertex exhibits a singularity at  $P^2 = 0$  then (10), or even (8), may be reconcilable with the phenomena of the strong interaction. However, whether that is truly the case will likely depend on details and therefore require *fine-tuning* in the theory.

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<sup>10</sup> A kindred study, Ref. [21], employs Ansätze for the ghost-gluon and 3-gluon vertices that exhibit particle-like singularities. Recall that no evidence for such behaviour is observed in lattice-QCD estimates of the 3-gluon vertex [8].

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## References

- [1] C. D. Roberts, “Nonperturbative QCD with Modern Tools”, nucl-th/9807026.
- [2] P. Maris, C. D. Roberts and P. C. Tandy, Phys. Lett. B **420** (1998) 267.
- [3] P. Maris and C. D. Roberts, Phys. Rev. C **56** (1997) 3369.
- [4] M. R. Pennington, “Calculating hadronic properties in strong QCD”, hep-ph/9611242.
- [5] N. Brown and M. R. Pennington, Phys. Rev. D **39** (1989) 2723.
- [6] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22** (1980) 2542.
- [7] M. R. Frank, Phys. Rev. C **51** (1995) 987.
- [8] C. Parrinello, Phys. Rev. D **50** (1994) R4247.
- [9] P. Jain and H. J. Munczek, Phys. Rev. D **48** (1993) 5403; P. C. Tandy, Prog. Part. Nucl. Phys. **39** (1997) 117; M. A. Pichowsky and T.-S. H. Lee, Phys. Rev. D **56** (1997) 1644; M. A. Ivanov, Yu. L. Kalinovsky, P. Maris and C. D. Roberts, Phys. Rev. C. **57** (1998) 1991; P. Maris, C.D. Roberts and S. Schmidt, Phys. Rev. C **57** R2821 (1998).
- [10] A. Bender, C. D. Roberts and L. v. Smekal, Phys. Lett. B **380** (1996) 7; C. D. Roberts, in *Quark Confinement and the Hadron Spectrum II*, edited by N. Brambilla and G. M. Prosperi (World Scientific, Singapore, 1997), pp. 224-230.
- [11] U. Häbel, R. Könnig, H.-G. Reusch, M. Stingl and S. Wigard, Z. Phys. A **336** (1990) 423; U. Häbel, R. Könnig, H.-G. Reusch, M. Stingl and S. Wigard, Z. Phys. A **336** (1990) 435.
- [12] F. T. Hawes, C. D. Roberts and A. G. Williams, Phys. Rev. D **49** (1994) 4683.
- [13] A. Bender and R. Alkofer, Phys. Rev. D **53** (1996) 446.
- [14] A. A. Natale and P. S. Rodrigues da Silva, Phys. Lett. B **392** (1997) 444.
- [15] P. Marenzoni, G. Martinelli and N. Stella, Nucl. Phys. B **455** (1995) 339.
- [16] J. M. Cornwall, Phys. Rev. D **26** (1982) 1453.
- [17] D. C. Curtis and M. R. Pennington, Phys. Rev. D **42** (1990) 4165.
- [18] UKQCD Coll. (Derek B. Leinweber, *et al.*) Phys. Rev. D **58** (1998) 031501.

- [19] D. Atkinson and P. W. Johnson, Phys. Rev. D **37** (1988) 2296; C. D. Roberts and B. H. J. McKellar, Phys. Rev. D **41** (1990) 672; H. J. Munczek and D. W. McKay, Phys. Rev. D **42** (1990) 3548.
- [20] D. Atkinson and J. C. R. Bloch, Mod. Phys. Lett. A **13** (1998) 1055.
- [21] L. v. Smekal, A. Hauck and R. Alkofer, “A solution to coupled Dyson-Schwinger equations for gluons and ghosts in Landau gauge”, hep-ph/9707327.